# DYNAMIC INSTABILITY IN HIGH-RISE STEEL STRUCTURES SUBJECTED TO STRONG GROUND MOTIONS: A REVIEW

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**Abstract**

It is well known that repeated excursions into the inelastic range of deformation of a structure in response to strong ground motions might cause single directional accumulated permanent deformations in the structure. It is also pointed out that, when high-rise buildings are subjected to very strong ground motions, drifting takes place only at the lower stories of the building, where the influence of the P-Delta effect is significant.

Unclear is the actual safety margin of structures designed in accordance with strong-column-weak-beam requirements of recent code provisions against dynamic instability because this effect is not properly considered in modern design provisions.

Thus, this paper provides a review of studies on dynamic instability in high-rise steel structures that is attributed to P-Delta effect.

*Keywords* : dynamic instability, P-Delta effect, long-period ground motion, structural safety, seismic analysis

## 1. Introduction

On 11 March 2011, the Great East Japan Earthquake caused massive damage over a wide area of eastern Japan. In particular, the severe damage caused by the tsunami received much attention. As for ground motions, it can be said that recent research in earthquake engineering contributed to mitigating the damage. Yet problems remain in regard to vibrations induced by ground motions.
Long-period ground motion and expected severe seismic events (such as a future Nankai earthquake) are issues that have recently attracted widespread public attention in Japan.
It is well known that repeated excursions into the inelastic range of deformation of a structure in response to strong ground motions might cause single directional accumulated permanent deformations in the structure. It is also pointed out that, when high-rise buildings are subjected to very strong ground motions, drifting or ratcheting takes place only at the lower stories of the building, where the influence of the P-Delta effect is significant.

Unclear is the actual safety margin of structures designed in accordance with strong-column-weak-beam requirements of recent code provisions against dynamic instability because this effect is not properly considered in modern design provisions.

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Thus, the objective of this paper is to provide a review of the literature regarding single directional drifting and dynamic instability of building structures subjected to severe ground motions.

## 2. Accumulation of inelastic deformation in one direction

Jennings and Husid (1968) pointed out that collapse would occur if the post-yield stiffness considering P-Delta effect is negative value by using a single-degree-of-freedom (SDOF) system (Fig. 1).

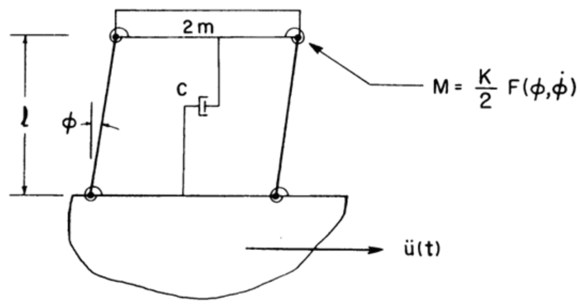


*Figure 1. SDOF model for studying gravity effects [Jennings and Husid (1968)]*

Sun, Berg and Hanson (1973) defined the P-Delta effect parameter as the ratio of the reduction of the lateral stiffness due to P-Delta effect to the initial stiffness neglecting gravity effect in an SDOF structure, which is also referred to as the gravity effect parameter or the stability coefficient, and stated that the P-Delta effect parameter, the maximum earthquake input energy, and the yield displacement are directly related to collapse of the structure.

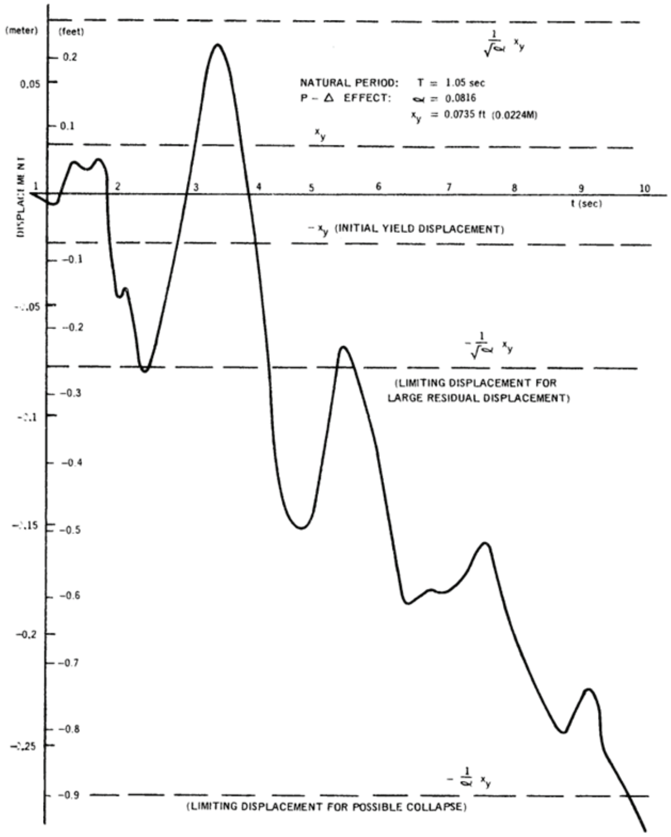


*Figure 2. Dynamic response of single-degree inelastic system to earthquake input [Sun, Berg and Hanson (1973)]*

As shown in Fig. 2, the response displacement of a single-degree elasto-plastic system of the slip type subjected to a ground motion tends not to return to the zero displacement position after the displacement exceeds the "limiting displacement for large residual displacement". The displacement also tends to increase without bound after it exceeds "the limiting displacement for possible collapse".

As for multi-degree-of-freedom (MDOF) system, Akiyama (1985) stated that the negative post-yield stiffness in a single story leads to damage concentration in the story. Uetani and Tagawa (1996) pointed out that the critical height of the deformation concentration region can be predicted by the Euler buckling of the long column having the effective buckling length of the region (Fig. 3).

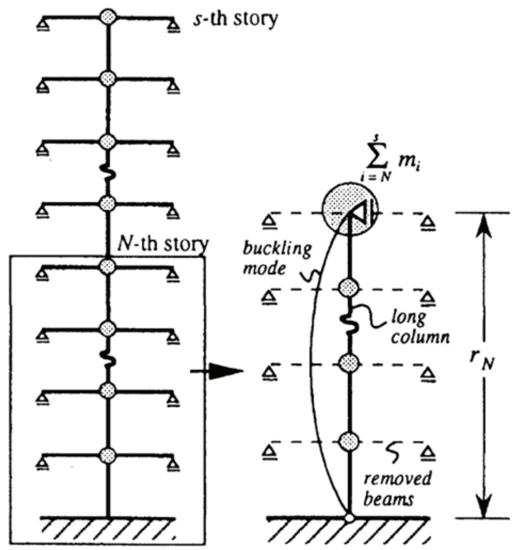


*Figure 3. Equivalent frame for predicting the critical height of the deformation concentration region [Uetani and Tagawa (1996)]*

Bernal (1998) pointed out that the necessary condition for an MDOF structure to be dynamically unstable is that at least one of the eigenvalues of the momentary stiffness matrix turns to be negative. Let the incremental first-order tangent stiffness and the geometric stiffness of the MDOF structure $\mathbf{K}_t$ and $\mathbf{K}_g$, respectively, relationship between incremental load $\Delta \boldsymbol{L}$ and the corresponding incremental displacement $\Delta \boldsymbol{u}$ during the infinitesimal time interval $\Delta t$ is represented by the following equation:

$$(\mathbf{K}_t - \mathbf{K}_g)\Delta \boldsymbol{u} = \Delta \boldsymbol{L} \tag{1}$$

The eigenvalues for the incremental stiffness matrix of the MDOF system can be obtained by solving the following equation:

$$(\mathbf{K}_t - \mathbf{K}_g)\boldsymbol{\phi} = \lambda \boldsymbol{\phi} \tag{2}$$

where, $\lambda_i$ and $\boldsymbol{\phi}_i$ are defined as $i$th eigenvalue and eigenvector, respectively.

An alternative formulation of the condition for static instability is obtained as follows:

$$\mathbf{K}_t \boldsymbol{\varphi} = \eta \mathbf{K}_g \boldsymbol{\varphi} \tag{3}$$

The lowest eigenvalue $\eta_b$ and the corresponding eigenvector $\boldsymbol{\varphi}_b$ for Eq. (3) respectively represent the scaling factor of the vertical load and mode vector for which equilibrium bifurcates. Obviously, $\eta_b < 1$ implies instability. A comparison of Eqs. (2) and (3) shows that when $\lambda = 0$, $\eta_b = 1$ and $\boldsymbol{\phi}_1 = \boldsymbol{\varphi}_b$. This implies, as Uetani and Tagawa (1996) pointed out, that displacement mode shape induced by dynamic instability is closely related to static buckling mode, as well as that the mechanism shape plays key role to the strength of the structure against the dynamic instability (Fig. 4).

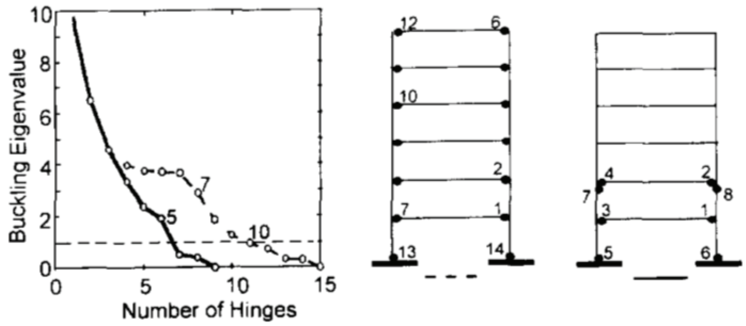


*Figure 4. Buckling eigenvalue* $\eta_b$ *as a function of the distribution of plastic hinges*
*[Barnal (1998)]*

A comprehensive review on accumulation of inelastic deformation in a single direction is given by Villaverde (2007).

### 3. Drifting at lower stories of building

Uetani and Tagawa (1996, 1998) pointed out that, when high-rise buildings are subjected to very strong ground motions, drifting takes place only at the lower stories of the building, where the influence of the P-Delta effect is significant, even when the building is designed in accordance with the strong-column-weak-beam concept.

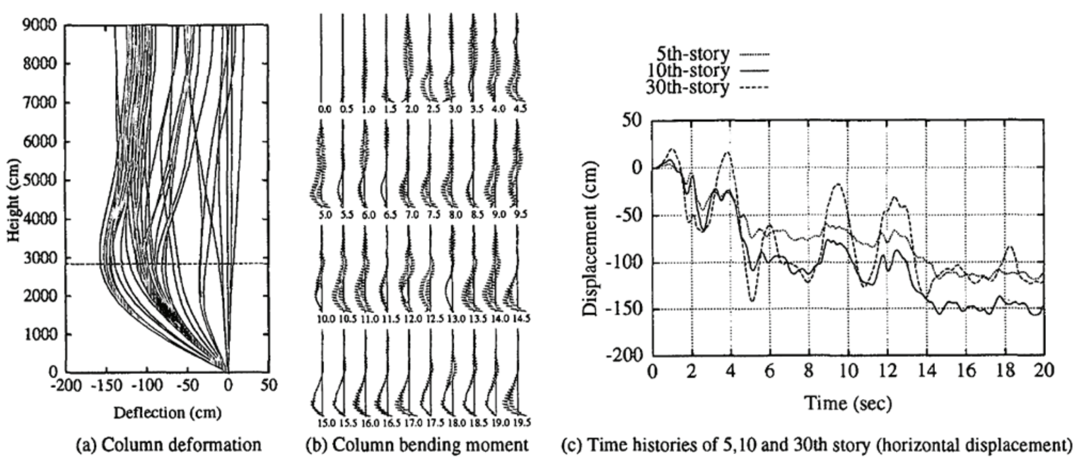


*Figure 5. Results of seismic response analysis (El Centro 1940 NS, PGV = 1.5m/s)*
*[Uetani and Tagawa (1996)]*

Time history analysis of thirty story fishbone-shaped frame was conducted, using three historical ground motions, El Centro 1940 NS, Taft 1952 EW, and Hachinohe 1968 NS, whose PGVs were scaled such that they were 1.5 m/s. The result for El Centro 1940 NS is shown in Fig. 5. Fig. 5(a) and 5(b) respectively show column deformation and heightwise distribution of column bending moment at the interval of 0.5 s. The dashed horizontal line in Fig. 5(a) represents the critical height of the deformation concentration region predicted by Euler buckling (Fig. 3). Time histories of 5th, 10th, and 30th story displacements relative to the ground shown in Fig 5(c) observe accumulation of displacements in a single direction.

Guputa and Krawinkler (2000a, 2000b) reported similar results by conducting time history analyses on the basis of finite element method using beam, column, and panel elements. Yamazaki and Endo (2000) studied the relationship between the intensity of ground motions and the magnitude of residual displacements cased by the dynamic instability by using the stability coefficient on the basis of seismic energy balance.

## 4. Condition for Dynamic Instability

Uetani and Tagawa (1998) pointed out that the condition for suppressing the deformation concentration in dynamic response is to keep the lowest eigenvalue of the tangential stiffness matrix positive, and illustrated the effectiveness of the proposed condition by parametric analyses where the post-yield stiffnesses of beams are used as parameters.

Bernal (1998) pointed out that the necessary condition for dynamic instability is that the lowest eigenvalue of the tangential stiffness matrix is negative, which implies exactly the same thing as Uetani and Tagawa (1998) pointed out.

Guputa and Krawinkler (2000b) pointed out that the dynamic instability unlikely occurs if the response roof displacement is smaller than the minimum displacement where the curve of normalized base-shear versus roof displacement turns to be a negative slope.

Tagawa, Macrae and Lowes (2007) defined three types of dynamic stability coefficients:

$$r_i(t) = \frac{\boldsymbol{\phi}_i(t)^T \mathbf{K}(t) \boldsymbol{\phi}_i(t)}{\boldsymbol{\phi}_{0,i}^T \mathbf{K}_0 \boldsymbol{\phi}_{0,i}} \tag{4}$$

$$\theta_i(t) = -\frac{\boldsymbol{\phi}_i(t)^T \mathbf{K}_g(t) \boldsymbol{\phi}_i(t)}{\boldsymbol{\phi}_{0,i}^T \mathbf{K}_0 \boldsymbol{\phi}_{0,i}} \tag{5}$$

$$r_{\mathrm{net},i}(t) = r_i(t) - \theta_i(t) \tag{6}$$

where, $r_i(t), \theta_i(t)$, and $r_{\mathrm{net},i}(t)$ are material, geometric, and net dynamic stability coefficients for the $i$ th eigenmode, respectively. $\mathbf{K}_0$ is the initial elastic stiffness matrix. $\mathbf{K}(t)$ and $\mathbf{K}_g(t)$ are the momentary tangential stiffness and geometric stiffness matrices at the time $t$, respectively. $\boldsymbol{\phi}_{0,i}$ and $\boldsymbol{\phi}_i$ respectively are the $i$th eigenvectors for the initial elastic stiffness matrix and the momentary tangent matrix, which are normalized such that:

$$\frac{\boldsymbol{\phi}_{0,i}(t)^T \mathbf{M1}}{\boldsymbol{\phi}_{0,i}^T \mathbf{M} \boldsymbol{\phi}_{0,i}} = \frac{\boldsymbol{\phi}_i(t)^T \mathbf{M1}}{\boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_i} = 1 \tag{7}$$

where, $\mathbf{M}$ and $\mathbf{1}$ are the mass matrix and the influence coefficient vector, respectively. The necessary condition for the dynamic instability is at least of the dynamic stability coefficients $r_{\mathrm{net},i}(t)$ turns to be negative.

## 5. Seismic Response of high-rise buildings subjected to long-period ground motions

Only a few research works addressed dynamic instability of high-rise buildings subjected to long-period ground motions. Such research dates back at least to the investigation of the 22-story Pino Suarez complex building (Fig. 6) which collapsed in the 1985 Michoacan, or Mexico City Earthquake (Osteraas and Krawinkler 1989, Ger *et al*. 1993).

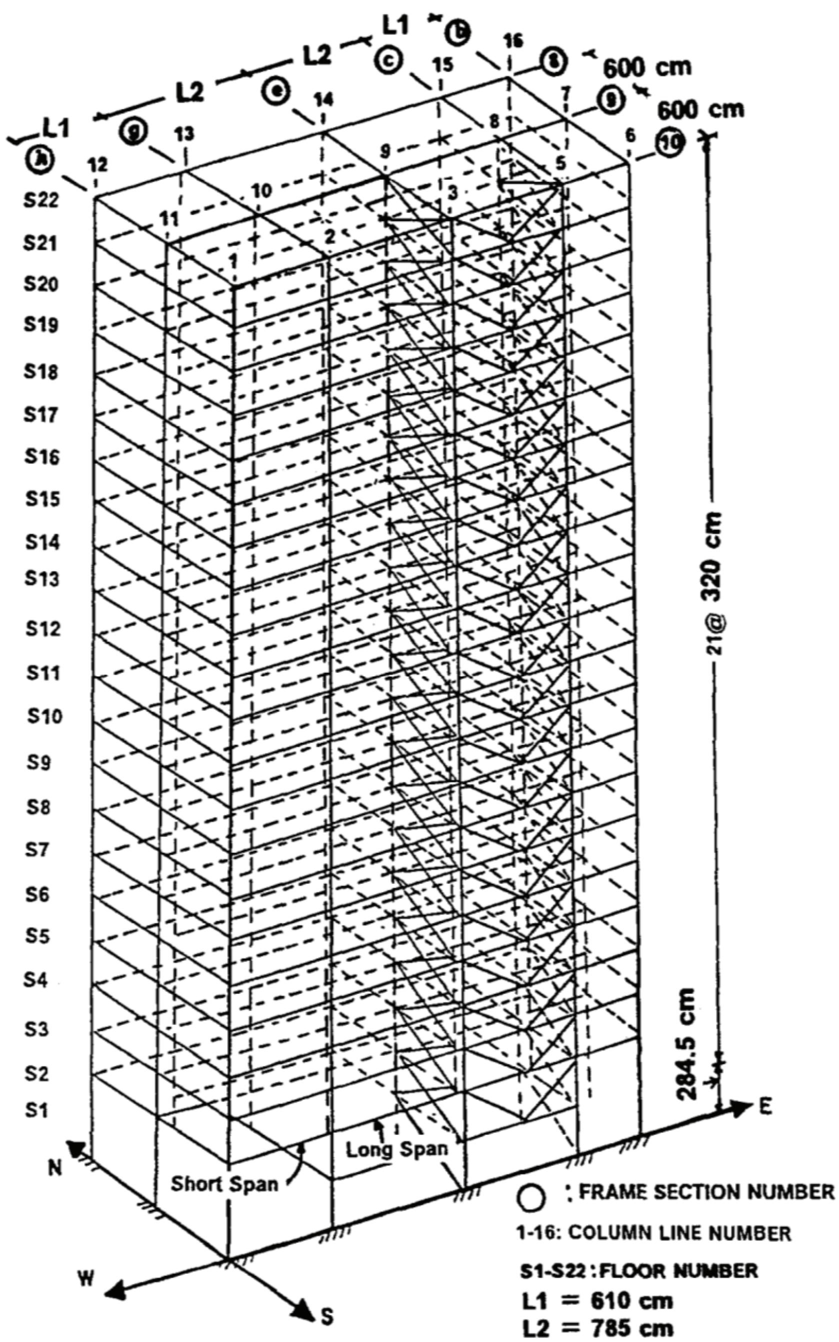


*Figure 6. Configuration of Pino Suarez Building [Ger et al. (1993)]*

Each research reported that a yield mechanism that is attributed to column yielding occurred in a certain lower story lead to the collapse. In their analyses, buckling of braces, local buckling of columns, and the P-Delta effect are taken into account. The analysis by Osteraas and Krawinkler (1989) observes single directional accumulated deformations which might be attributed to the P-Delta effect. Ger *et al.* (1993) compared the analyses with and without P-Delta effect, and pointed out that the collapse behavior was not simulated by the analysis without P-Delta effect. These papers suggest us that a symmetric structural plan in order to avoid torsional motions, strong connections, enhanced ductility in the girders, and strong columns for not having plastic hinges and local buckling developed in the columns are important to avoid collapse of structures.

Yamazaki and Minami (AIJ 2007) conducted analyses regarding single directional accumulated deformations by using realistic steel high-rise building models subjected to hypothetical long-period ground motions, in which, however, the analytical models and ground motions they used are limited.

Araki *et al*. (2011) employed 9 high-rise steel moment resisting frame (SMRF) models in accordance with the Japanese design practice in the 1980s. Table 1 lists the structural properties of the 9 SMRF models. For example, the model name "SMRF20-0.2" indicates that $N$ = 20 and $C_0$ = 0.2, where $N$ and $C_0$ are the number of stories and standard base shear coefficient, respectively. $C_B$, $C_U$, $T_1$, and $H$ are the design base shear, the base shear at the ultimate strength, the first fundamental period of the model, and the height of the model, respectively. $\Theta$ is the stability coefficient. The hardening parameter $\alpha$ is defined as the ratio of the post-yield story stiffness at the ultimate strength to the initial elastic story stiffness. The subscripts NPD and PD indicate that the hardening parameters are obtained by the pushover analysis neglecting and considering the P-Delta effect, respectively.

Table 1.

REPRESENTATIVE VALUES FOR DESIGN OF SMRF MODELS.

| SMRF | 20-0.2 | 20-0.3 | 20-0.4 | 30-0.2 | 30-0.3 | 30-0.4 | 40-0.2 | 40-0.3 | 40-0.4 |
|---|---|---|---|---|---|---|---|---|---|
| $N$ | 20 | 20 | 20 | 30 | 30 | 30 | 40 | 40 | 40 |
| $C_0$ | 0.2 | 0.3 | 0.4 | 0.2 | 0.3 | 0.4 | 0.2 | 0.3 | 0.4 |
| $C_B$ | 0.08 | 0.12 | 0.16 | 0.04 | 0.08 | 0.12 | 0.03 | 0.06 | 0.09 |
| $C_U$ | 0.15 | 0.20 | 0.27 | 0.09 | 0.13 | 0.18 | 0.08 | 0.11 | 0.15 |
| $C_U/C_B$ | 1.92 | 1.68 | 1.66 | 2.25 | 1.63 | 1.50 | 2.67 | 1.83 | 1.67 |
| $T_1$ (sec) | 3.06 | 2.56 | 2.18 | 4.59 | 3.78 | 3.29 | 5.95 | 4.90 | 4.23 |
| $H$ (m) | 81 | 81 | 81 | 121 | 121 | 121 | 161 | 161 | 161 |
| $T_1$ (sec)/$H$ (m) | 0.038 | 0.032 | 0.027 | 0.038 | 0.031 | 0.027 | 0.037 | 0.030 | 0.026 |
| $\Theta$ (%) | 4.48 | 3.24 | 2.46 | 7.90 | 4.15 | 2.83 | 8.53 | 4.33 | 2.91 |
| $\alpha_{NPD}$ (%) | 1.65 | 1.77 | 1.87 | 1.26 | 1.38 | 1.45 | 1.31 | 1.58 | 1.42 |
| $\alpha_{PD}$ (%) | −1.49 | −0.60 | 0.08 | −2.55 | −1.38 | −0.76 | −2.90 | −0.82 | −0.73 |

[Araki *et al.* (2011)]

Two synthetic long-period ground motions, C-SAN EW and KK-WOS EW as well as three historical ground motions, El Centro 1940 NS, Taft 1952 EW, and Hachinohe 1968 NS are employed as the input ground motions (Fig. 7). The two synthetic ground motions are predicted to occur in Nagoya and Osaka, respectively, caused by hypothetical Tonankai and Nankai earthquakes, and are identified to have the largest influences on the response of high-rise buildings among the synthetic ground motions generated by the committee of Architectural Institute of Japan.

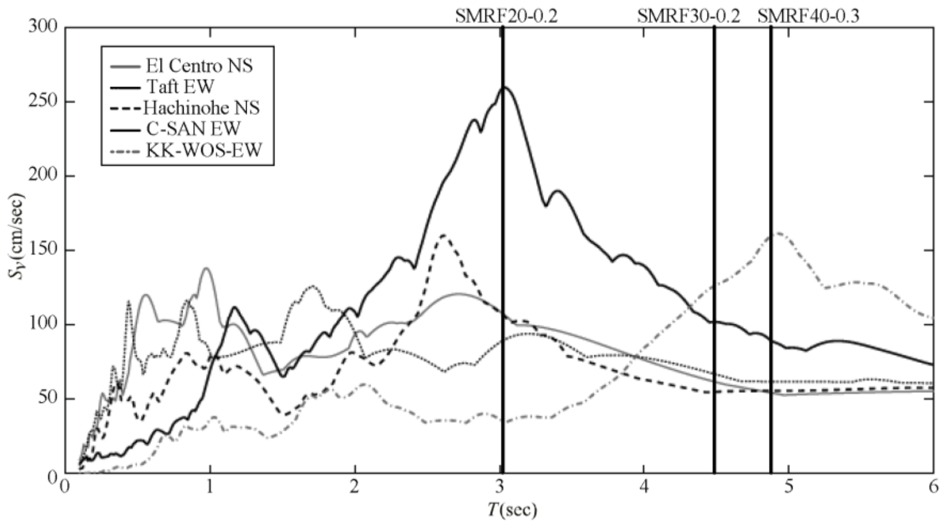


*Figure 7. Velocity response spectra of input ground motions (damping ratio = 5 %) [Araki et al. (2011)]*

Among the 9 SMRF models, drifting took place in the response of SMRF20-0.2 to C-SAN EW, and the responses of SMRF30-0.2 and SMRF40-0.3 to KK-WOS EW. The natural periods of these models are shown by the bold vertical lines in Fig. 7. One can see from Fig. 7 that drifting took place when the natural periods of the SMRF models are close to the dominant periods of the long-period ground motions.

## 6. Conclusion

This paper provided a review on the literature regarding the single directional accumulated deformations and dynamic instability of high-rise steel structures. Studies using the high-rise steel structure models considering the P-Delta effect showed that, dynamic instability might occur even if the structure is designed in accordance with the recent design provisions.

Thus, it can be said that the P-Delta effect must be taken into account in design practice for high-rise steel structures to understand the actual safety margins.

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